Mesirow

Which asset classes are most beneficial in a portfolio?

Traditional mean variance optimization (MVO) is one type of asset allocation modeling that is an example of single period modeling.[1] It is used to determine efficient frontier allocations to different asset classes ranging from minimum to maximum standard deviation levels. So, rather than varying based on time horizon like target date funds (TDFs), it focuses on different risk levels for one specific horizon period.

Keith Gustafson, CFA
Managing Director of Asset Allocation and Retirement Income

Christopher O'Neill, PhD, CFA, CFP®, FRM®, ChFC®
Managing Director, Chief Investment Officer and Director of Quantitative Research

The key inputs for MVO are return, risk (standard deviation or variance) and correlation. While most investors are focused on return, the drivers of effective asset combinations are volatility and correlation (i.e., covariance). Less volatility means less variation in the average return that we expect from one period to another, while correlation indicates how the different asset returns typically move in relation to each other. To diversify a portfolio, assets that move differently from each other are the most beneficial. A lower correlation coefficient between two assets means less related movement between them. If an asset has a higher return and lower volatility and correlation with other assets in a portfolio relative to a competing asset, then it will dominate the less efficient asset in a portfolio context.

We utilize mean variance optimization analysis in order to demonstrate how different asset classes push out the efficient frontier opportunity set. As the efficient frontier moves 'northwest' (upward and to the left of a return vs. risk plot), the portfolios along the frontier produce higher expected return at lower expected volatility. A higher efficient frontier means that there is a portfolio combination that produces higher return at the same level of risk or lower risk at the same level of return relative to any point on a lower efficient frontier.

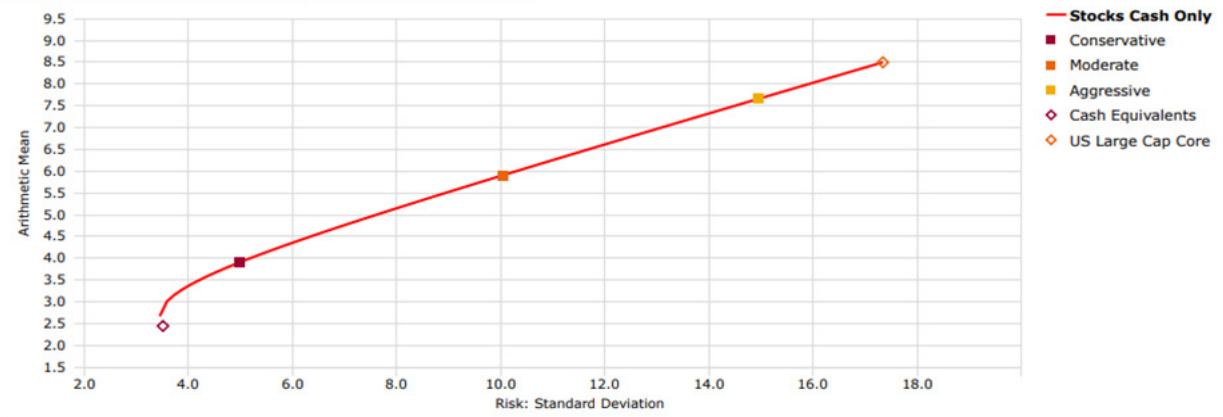
To facilitate our analysis, we create three different risk level portfolios at expected standard deviation levels of 5%, 10% and 15%, that we label as Conservative, Moderate and Aggressive. By comparing these portfolios of a constant expected risk level on each efficient frontier, we can see how the optimal combination of assets changes as the opportunity set of asset classes changes, as well as how the expected return improves for the same expected volatility level.

Baseline Portfolio: Cash equivalents and large cap US equity

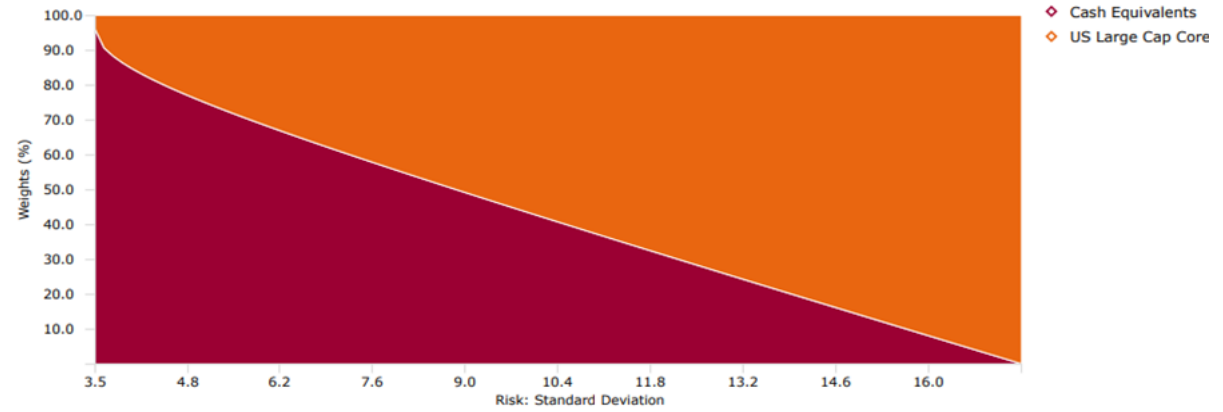
Figure 1 shows the simplest combination of assets as a starting point that consists of cash equivalents (US 30-day T-Bill) and large cap US stocks (Russell 1000 Index). In essence, this opportunity set is the heart of the Capital Asset Pricing Model (CAPM) and the Security Market Line (SML) derivation.[2] [3] In that framework, an investor can borrow or lend at the risk-free rate (cash equivalents) to invest in the maximum Sharpe Ratio portfolio (highest risk-adjusted return) on the efficient frontier to achieve the desired risk level. The opportunity set becomes a straight line at the point of tangency with the efficient frontier. In the SML derivation, the borrowing and lending rate is the same (cash equivalents), with systematic risk (Beta) defined by the stock market portfolio with a Beta of 1, and any investment opportunities evaluated relative to the SML. In practice, most investors (outside of hedge funds) are not levering portfolios up or down, any borrowing is not at the risk-free rate, and leverage is constrained and not unbounded. Instead, investors invest directly in asset classes in varying proportions, and that is how our analysis is structured, rather than a focus on a single maximum Sharpe Ratio portfolio on each frontier. Nevertheless, it can be useful to think about these efficient frontiers in the context of the underlying CAPM-framework.

Figure 1 shows both the efficient frontier consisting of the two assets, cash equivalents and stocks, along with the plots of the individual asset classes and the three different risk level portfolios. The horizontal axis represents expected standard deviation and the vertical axis displays expected return, both based on inputs from Mesirow Fiduciary Solutions' capital market assumptions. The bottom chart is an area graph that displays the combination of assets for every point along the efficient frontier. With the two-asset opportunity set it is largely a straight line, rather that a bowed curve, except at the lowest risk levels. Because of the low correlation between stocks and cash equivalents, quickly adding stocks improves the expected return without increasing risk.

FIGURE 1: **EFFICIENT FRONTIER: ARITHMETIC MEAN VS. STANDARD DEVIATION**



FRONTIER ASSETS



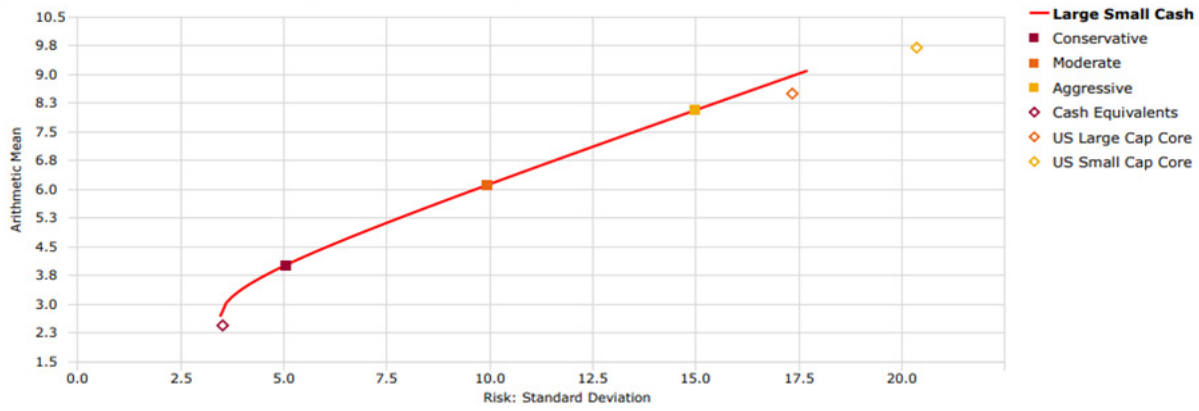
Source: MPI Stylus Data, Morningstar Asset Allocation Software and Mesirow Calculations and CMAs.

Expansion with small cap equity

The next efficient frontier in **Figure 2** shows what happens when small cap stocks (Russell 2000 Index) are added to the opportunity set. It is difficult to see the relative positioning of each frontier to the prior set, but one can assume that if the newly added asset(s) shows up in the area graph, then the efficient frontier has also shifted upward toward higher expected return for a given level of risk. In this asset opportunity set, we can observe that there is still a largely straight-line shifting from cash to stocks in aggregate and that the stocks allocation splits evenly between large cap and small cap. This is driven by a relative group constraint that we have in the MVO, where small cap cannot represent more than 50% of the total US stock weighting. The current market neutral weight of the large cap Russell 1000 Index in the total market Russell 3000 Index is 93%, while the small cap Russell 2000 Index has a weight of 7%. Therefore, allowing a 50% weight to small cap stocks is still allowing for substantial latitude.

Frontier portfolios, as mathematical constructs based upon covariance, at times can represent very concentrated portfolios that are not adequately diversified and are unstable with slight input changes. The goal of the analysis is to understand which asset classes tend to dominate and improve portfolio characteristics in a meaningful way but bounded by some practical diversification considerations. These boundaries are represented by group relative constraints and absolute constraints, in some cases. In Figure 2, all three assets are utilized at some point along the frontier. Later in the paper, we directly compare all of the frontiers and risk level portfolios to better quantify the impact of each sequential asset class included in the opportunity set.

FIGURE 2: **EFFICIENT FRONTIER: ARITHMETIC MEAN VS. STANDARD DEVIATION**



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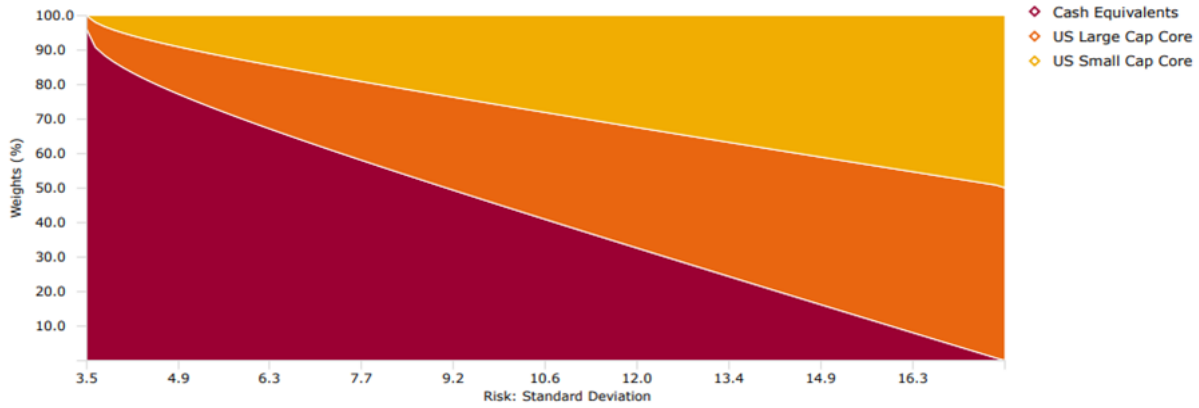
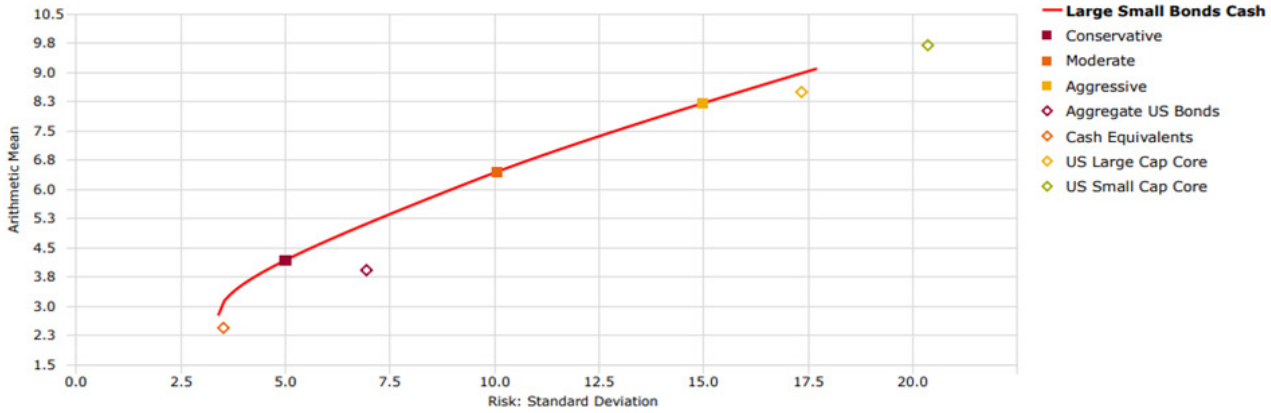


Source: MPI Stylus Data, Morningstar Asset Allocation Software and Mesirow Calculations and CMAs.

Integration of aggregate US bonds

The asset class opportunity set in **Figure 3** includes an additional fixed income asset class of aggregate US bonds (Bloomberg US Aggregate Bond Index) in addition to ultra-short duration cash equivalents. Relative to the prior efficient frontier, the large cap and small cap stock allocations do not appear to change meaningfully, but there is a clear transition from cash equivalents to aggregate bonds with increasing risk level and the bonds allocations are present even in the minimal risk portfolio at the far left of the area graph.

FIGURE 3: **EFFICIENT FRONTIER: ARITHMETIC MEAN VS. STANDARD DEVIATION**



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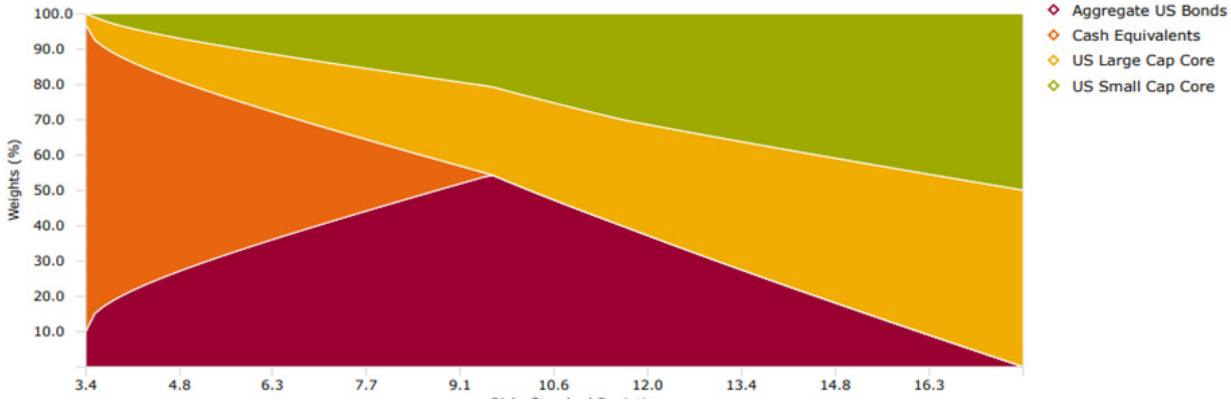


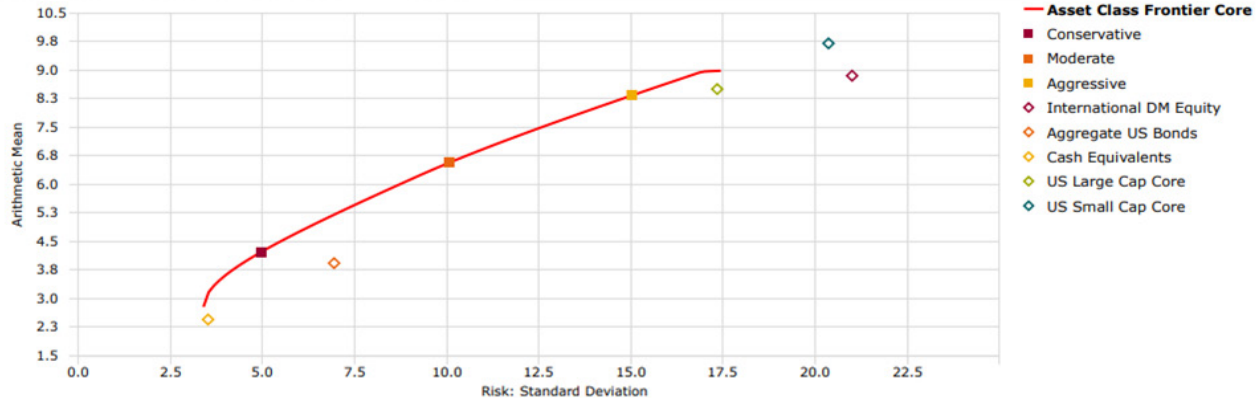
Source: MPI Stylus Data, Morningstar Asset Allocation Software and Mesirow Calculations and CMAs.

Core Asset Class opportunity set: Inclusion of international developed equity

The next efficient frontier set is what Mesirow refers to as the "Core Asset Class" opportunity set, which is generally what we desire to see as the minimal asset complexity for investment lineup selection and portfolio construction. In addition to cash equivalents, bonds and US large cap and small cap stocks, international developed market equity (MSCI EAFE Index) is added to complete the 'Core Asset Class' opportunity set. This turns the asset classes represented from that of a purely domestic construct to one that represents a global opportunity set that provides for international diversification. This asset class opportunity set is shown in **Figure 4**.

The market neutral weight of US equities in the global equity MSCI All Country World Index is currently just above a hefty 64%, while we allow the international equities to represent up to 50% of the total equity weight in the relative group constraints, which is well above the market neutral 36% weight. Interestingly, while the efficient frontier takes advantage of the international diversification benefit, it does not allocate anywhere close to the 50% constraint threshold. Instead, the allocation as a percentage of equities is closer to 30% or less on average. International allocations is a topic that we explore in depth in another Mesirow white paper[4] and that we also touch on later in this paper.

FIGURE 4: **EFFICIENT FRONTIER: ARITHMETIC MEAN VS. STANDARD DEVIATION**



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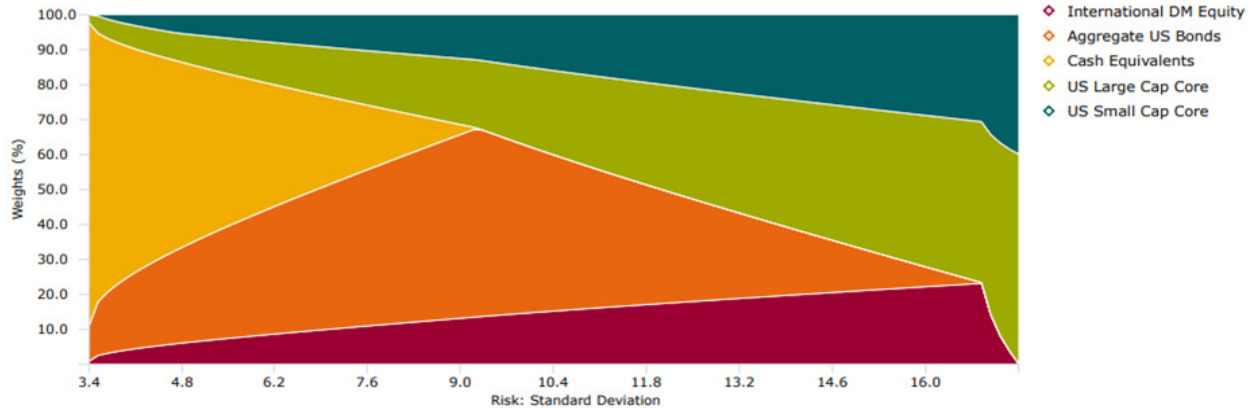


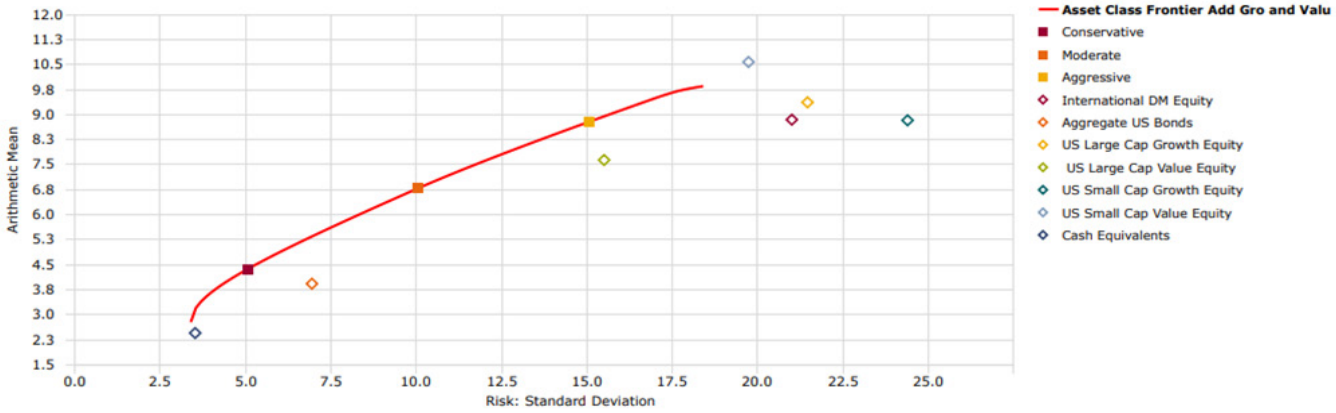
Source: MPI Stylus Data, Morningstar Asset Allocation Software and Mesirow Calculations and CMAs.

Equity style diversification: Growth versus value factors

Up to this point, we have found that the MVO efficient frontier likes to maximize small cap core equities up to the allowed constraint of 50%, which is significantly higher than market neutral weight for this asset class, while it prefers international developed market equities to a lesser extent, allocating below the market neutral weight for this asset class. **Figure 5** shows the allocation results when core US large cap and small cap equities are replaced with investment style options representing growth and value US large cap and small cap equities. When presented with this style opportunity set that we do not constrain, the results are interesting. The optimizer prefers primarily to allocate to large cap growth (Russell 1000 Growth Index) and small cap value (Russell 2000 Value Index). Large cap value (Russell 1000 Index) does receive some allocations across portfolios, while small cap growth (Russell 2000 Growth Index) is completely shut out of all efficient frontier portfolios. Further, the introduction of growth and value US equities to the asset class opportunity set slightly reduces the optimal allocation to international equities across the risk spectrum.

One of the apparent drivers of these allocation results is the fact that the correlations between large cap growth and small cap value are very low and the historical size premium (i.e., the extra return over the long term in after adjusting for the risk of small cap stocks relative to large cap) is most represented on the value side of the ledger. So, if small cap value is preferred, then large cap growth becomes the ideal complement.

FIGURE 5: **EFFICIENT FRONTIER: ARITHMETIC MEAN VS. STANDARD DEVIATION**



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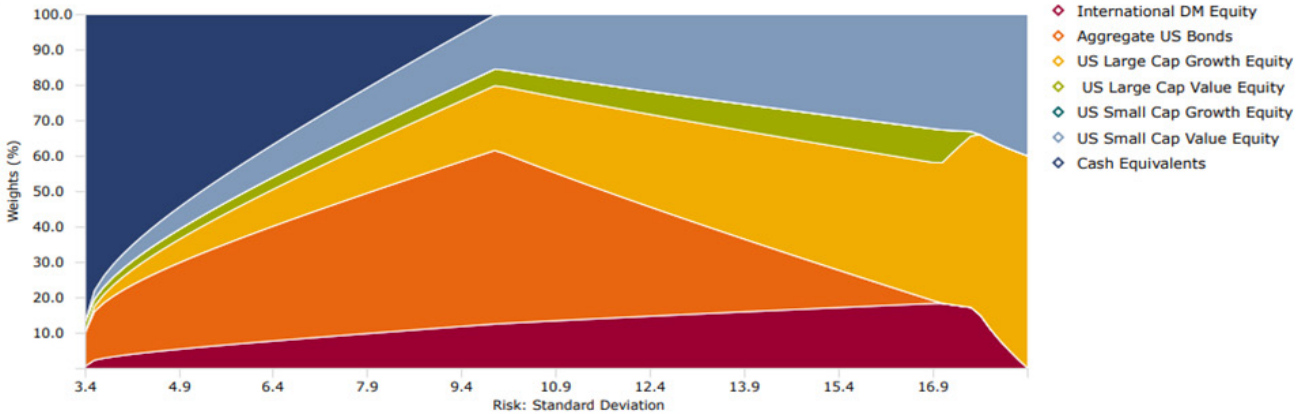


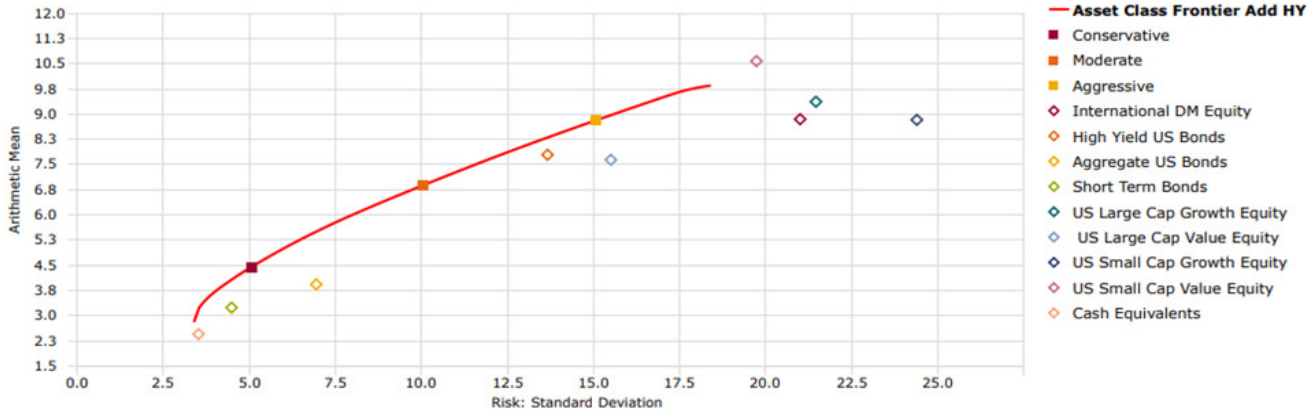
Source: MPI Stylus Data, Morningstar Asset Allocation Software and Mesirow Calculations and CMAs.

Additional fixed income segments: Short-term and high yield bonds

The previous two efficient frontier opportunity sets represented the addition of various equity asset classes, while **Figure 6** presents the addition of two fixed income asset classes in the form of short-term bonds (Bloomberg US 1–3 Year Government Bond Index) and high yield bonds (Bloomberg High Yield Bond Index). Not surprisingly, with respect to fixed income, the weighting on the efficient frontier depends on the relative standard deviation level. The cash equivalents give way to short-term bonds and then aggregate bonds in a fairly linear fashion as the volatility level increases.

High yield bonds, which carry more credit risk than many other bond types, are more of a hybrid between bonds and equities, which accordingly causes it to pull from both elements in the efficient frontier allocations. High yield allocations are present through the majority of the efficient frontier, except for the aggressive standard deviation levels.

FIGURE 6: **EFFICIENT FRONTIER: ARITHMETIC MEAN VS. STANDARD DEVIATION**



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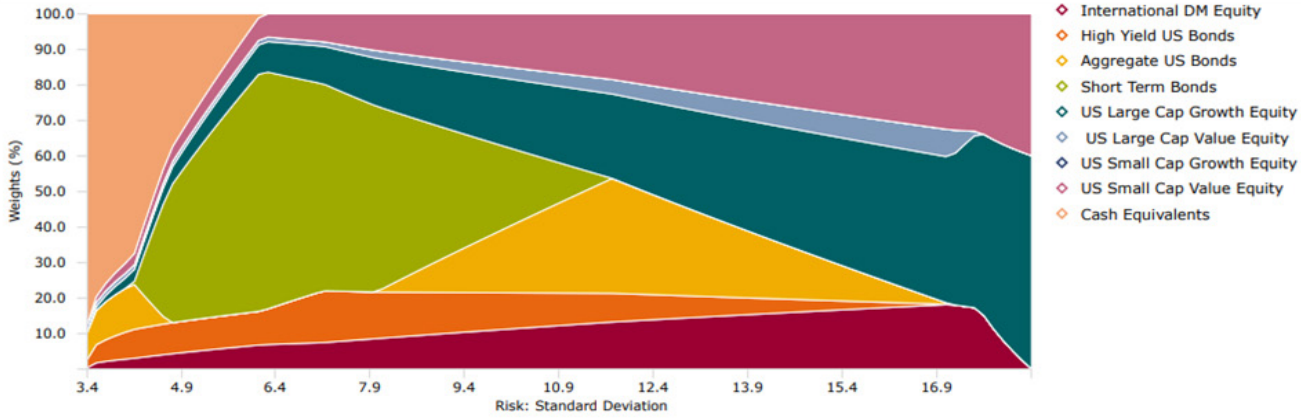


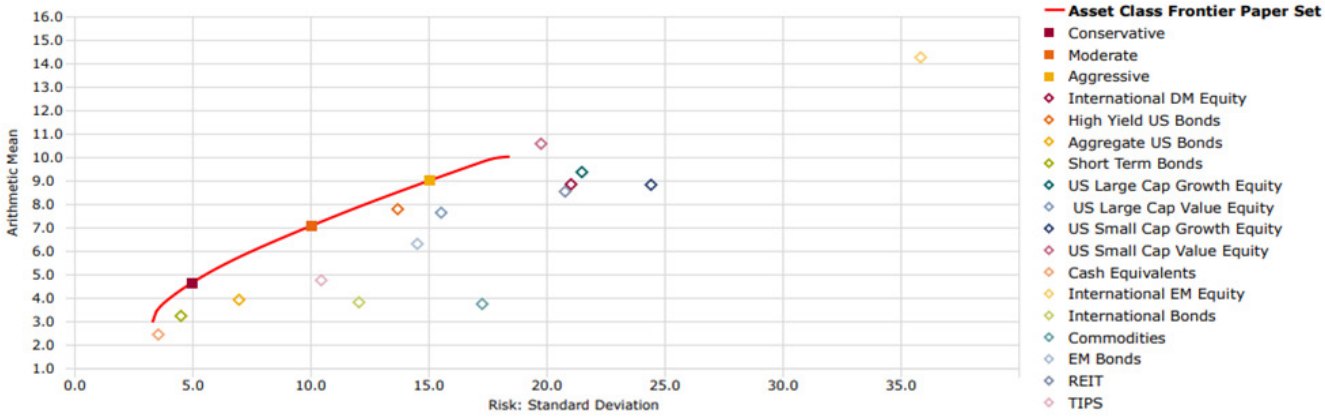
Source: MPI Stylus Data, Morningstar Asset Allocation Software and Mesirow Calculations and CMAs.

Supplemental asset classes: Inflation-linked and emerging market assets

Figure 7 presents the most complete asset class set, adding inflation-related assets, such as commodities (Bloomberg Commodity Index), TIPS (Bloomberg Global US TIPS Index), REITs (FTSE NAREIT Equity REITs Index), as well as international bonds (FTSE Non-US Dollar World Government Bond Index) and emerging market bonds (JP Morgan EMBI Plus Index), and finally emerging market equities (MSCI EM Index). Neither international bonds nor emerging market bonds are utilized by the optimizer at any point. All of the inflation-related assets are utilized at some point along the efficient frontier, although commodities are only utilized to a small extent in the lowest risk portfolio. The TIPS are utilized most heavily based upon the set of capital market assumptions in this analysis. Even though all the inflation-related assets are utilized in this analysis, scenario analysis that might include high sustained inflation scenarios could result in much higher allocations to these assets. This is a topic that we address in a different Mesirow white paper.[5]

While the international and emerging market bonds are not preferred, the emerging market equities asset class addition is attractive to the optimizer and increases the overall international equity allocation, even though it still does not approach the relative asset class constraint and remains well below the market neutral global weighting.

FIGURE 7: **EFFICIENT FRONTIER: ARITHMETIC MEAN VS. STANDARD DEVIATION**



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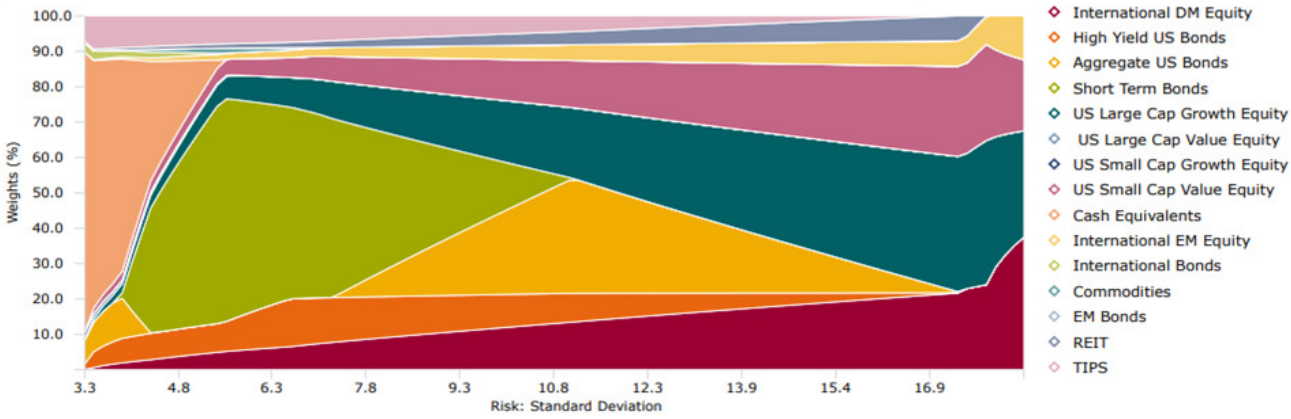
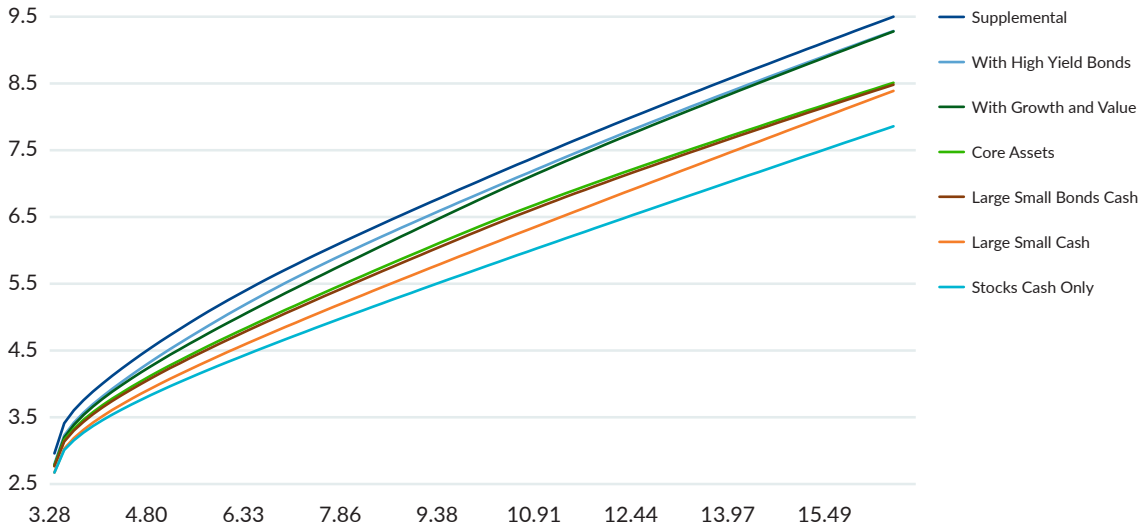


Source: MPI Stylus Data, Morningstar Asset Allocation Software and Mesirow Calculations and CMAs.

Comparative analysis of efficient frontier progressions

The previous analyses were stepwise in the process of including a more comprehensive asset class set, and while the area graphs display the frontier set of asset classes varying by risk level, an overall picture of the scale and quantification of improvement has been missing until now. **Figure 8** displays the efficient frontiers from each set in order to visualize the improvement in return for a given level of risk with the addition of asset classes. A bigger jump towards the 'northwest' in risk and return space represents a bigger improvement from the addition of those particular asset classes to the prior set. There are large jumps with the addition of small cap equities and then again with bonds, while the addition of international equities results in only a minor improvement. The addition of domestic style equities makes a big difference that scales larger with the risk level, while the addition of high yield and short-term bonds makes a slight difference mostly at lower risk levels. There is incremental improvement across risk levels with the addition of more international assets and inflation-linked assets.

FIGURE 8: ASSET CLASS OPPORTUNITY SETS MVO EFFICIENT FRONTIERS



Source: MPI Stylus Data, Morningstar Asset Allocation Software and Mesirow Calculations and CMAs.

Portfolio allocation outcomes and risk-return enhancements

Table 1 shows the efficient frontier progressions for the different risk level portfolios of Conservative, Moderate and Aggressive for each set. It is important to note that the core domestic equities are not competing, but instead are replaced with the growth and value equity options when they enter the frontier set in order to avoid the multicollinearity issue in an MVO framework.[6] Prior to growth and value, it is a core equity option and then growth and value thereafter.

Of some note, the international bonds and commodity assets make a showing in the supplemental asset class set in small amounts in only the Conservative portfolio. The international equity allocation declines with the addition of growth and value domestic equities but then recovers slightly with the addition of emerging market equities. High yield bonds pull from both fixed income and equity allocations where they are included. The TIPs and REITs appear to pull allocation also from equities and fixed income.

TABLE 1: **EFFICIENT FRONTIER MVO ALLOCATIONS**

		Fixed Income						Equities								Inflation-linked		
	Asset Class Set	Cash Equiv.	Short-Term Bonds	Agg Bonds	High Yield	Intl Bonds	EM Bonds	LC Stocks	LC Growth	LC Value	SC Stocks	SC Growth	SC Value	Intl DM Equity	EM Equity	TIPs	REITs	Comm.
CONSERVATIVE	Stocks/Cash	75.81						24.19										
	Add SmallCap	75.99						14.25			9.76							
	Add Bonds	51.70		28.14				12.62			7.54							
	Add International (Core Assets)	50.79		28.45				8.66			5.77			6.33				
	Add Growth and Value	52.31		25.39					7.04	2.92			6.64	5.71				
	Add High Yield/ ST Bonds	28.75	45.67		8.91				5.69	1.38			4.71	4.88				
	Supplemental Asset Class Set	15.05	50.92		7.85	0.89			5.05	0.63			3.78	4.00	1.33	8.30	1.05	1.14
MODERATE	Stocks/Cash	42.82						57.18										
	Add SmallCap	44.65						29.18			26.18							
	Add Bonds			50.91				26.17			22.92							
	Add International (Core Assets)			47.44				22.73			15.15			14.68				
	Add Growth and Value			48.04					18.76	4.80			15.71	12.70				
	Add High Yield/ ST Bonds		18.34	22.67	10.25				19.21	3.25			15.02	11.19				
	Supplemental Asset Class Set		14.34	23.27	9.40				17.30				11.54	11.80	3.93	5.22	3.20	
AGGRESSIVE	Stocks/Cash	13.83						86.17										
	Add SmallCap	15.46						43.07			41.46							
	Add Bonds			17.30				41.35			41.35							
	Add International (Core Assets)			12.26				40.07			26.71			20.96				
	Add Growth and Value			12.68					33.89	8.35			28.16	16.93				
	Add High Yield/ ST Bonds			11.85	2.96				34.90	6.40			27.53	16.35				
	Supplemental Asset Class Set			11.80	2.95				31.69				21.13	18.69	6.23	5.87	1.64	

Source: MPI Stylus Data, Morningstar Asset Allocation Software and Mesirow Calculations and CMAs.

Table 2 quantifies the incremental improvement for each different risk level portfolio in terms of return improvement and risk-adjusted return defined by the Sharpe Ratio. For the Conservative portfolio, the addition of bonds and the supplemental assets improved the risk-adjusted return the most. For the Moderate and Aggressive portfolios, the addition of small cap and growth and value helped the most in terms of that risk and return efficiency.

TABLE 2: EFFICIENT FRONTIER MVO ALLOCATIONS

	Asset Class Set	Incremental gain	Return	Sharpe Ratio	Incremental gain
CONSERVATIVE	Stocks/Cash		3.91	0.78	
	Add SmallCap	0.11	4.02	0.80	0.02
	Add Bonds	0.16	4.18	0.84	0.04
	Add International (Core Assets)	0.04	4.22	0.85	0.01
	Add Growth and Value	0.15	4.37	0.86	0.01
	Add High Yield/ST Bonds	0.08	4.45	0.88	0.02
	Supplemental Asset Class Set	0.21	4.66	0.94	0.06
MODERATE	Stocks/Cash		5.91	0.59	
	Add SmallCap	0.20	6.11	0.62	0.03
	Add Bonds	0.34	6.45	0.64	0.02
	Add International (Core Assets)	0.12	6.57	0.65	0.01
	Add Growth and Value	0.23	6.80	0.68	0.03
	Add High Yield/ST Bonds	0.09	6.89	0.69	0.01
	Supplemental Asset Class Set	0.19	7.08	0.71	0.02
AGGRESSIVE	Stocks/Cash		7.66	0.51	
	Add SmallCap	0.40	8.06	0.54	0.03
	Add Bonds	0.15	8.21	0.55	0.01
	Add International (Core Assets)	0.12	8.33	0.56	0.01
	Add Growth and Value	0.46	8.79	0.58	0.02
	Add High Yield/ST Bonds	0.03	8.82	0.59	0.01
	Supplemental Asset Class Set	0.20	9.02	0.60	0.01

Source: MPI Stylus Data, Morningstar Asset Allocation Software and Mesirow Calculations and CMAs.

Table 3 elucidates how the overall allocation to specific assets changes. The overall equity allocation varies with the addition of new assets, declining and then increasing and then declining again. The internal equity allocation is defined as the percentage of total equities rather than an absolute percentage allocation. That also declines and then increases again with various asset inclusion. The alternative allocation percentage is an absolute percentage allocation and is only relevant for the supplemental asset set. It is included to demonstrate the meaningful allocation to these assets in all three risk-level portfolios.

TABLE 3: **EFFICIENT FRONTIER MVO ALLOCATIONS**

	Asset Class Set	Equity %	Intl Equity %	Alt Total %
CONSERVATIVE	Stocks/Cash	75.81		
	Add SmallCap	75.99		
	Add Bonds	51.70		28.14
	Add International (Core Assets)	50.79		28.45
	Add Growth and Value	52.31		25.39
	Add High Yield/ST Bonds	28.75	45.67	
	Supplemental Asset Class Set	15.05	50.92	
MODERATE	Stocks/Cash	75.81		
	Add SmallCap	75.99		
	Add Bonds	51.70		28.14
	Add International (Core Assets)	50.79		28.45
	Add Growth and Value	52.31		25.39
	Add High Yield/ST Bonds	28.75	45.67	
	Supplemental Asset Class Set	15.05	50.92	
AGGRESSIVE	Stocks/Cash	75.81		
	Add SmallCap	75.99		
	Add Bonds	51.70		28.14
	Add International (Core Assets)	50.79		28.45
	Add Growth and Value	52.31		25.39
	Add High Yield/ST Bonds	28.75	45.67	
	Supplemental Asset Class Set	15.05	50.92	

Source: MPI Stylus Data, Morningstar Asset Allocation Software and Mesirow Calculations and CMAs.

Conclusion: Diversification and the role of supplemental assets

What is surprising is how many asset classes are beneficial in portfolio formation even when utilizing a purely mathematical MVO framework. MVO can produce narrow outcomes if unconstrained, but reasonable relative constraints can inform us about relative advantages of various asset classes.

As noted in the beginning of this paper, the key inputs for MVO are return, risk (standard deviation or variance) and correlation. It is widely recognized that modeling volatility and correlation is easier than modeling return, which can be more time-varying, but any forward-looking inputs will have errors. We drive forward by looking in the rearview mirror, but with long-term data over many capital market regimes and with market expectations of inflation and interest rates, we can do a reasonable job of approximating outcomes over long horizons and evaluate the relative efficiency and interaction of different asset classes. In the face of uncertainty, both common sense and practical experience teach us that diversification beats concentration. Even the MVO analyses that we have utilized here that pre-suppose mathematical certainty clearly demonstrate a preference for asset class diversification that produces better and more stable portfolio outcomes with reasonable constraints. The analyses demonstrate that the Mesirow Core Asset Class set for lineup construction produces solid and diversified portfolio outcomes but also show that the addition of each additional supplemental asset class can improve the opportunity set.

About Mesirow

Mesirow is an independent, employee-owned financial services firm founded in 1937. Headquartered in Chicago, with offices around the world, we serve clients through a personal, custom approach to reaching financial goals and acting as a force for social good. With capabilities spanning Private Capital & Currency, Capital Markets & Investment Banking, and Advisory Services, we invest in what matters: our clients, our communities and our culture.

Mesirow Fiduciary Solutions creates institutional investment solutions for the retirement and health savings marketplace, supporting plan sponsors, financial advisors, recordkeepers and administrators, trust companies and insurers.

To learn more about how Mesirow can help you, please contact us at fiduciaryinquiries@mesirow.com or visit mesirow.com/fiduciarysolutions.

Bloomberg Commodity Index: A benchmark index that tracks the performance of a broad range of commodity futures contracts across energy, precious metals, industrial metals, livestock, and agricultural commodities.

Bloomberg Global US TIPS Index: This index measures the performance of US Treasury Inflation Protected Securities (TIPS) that are investment-grade, inflation-linked governmental debt, and denominated in US dollars.

Bloomberg US Aggregate Bond Index: The index is a composite of four major sub-indexes: US Government Index; US Credit Index; US Mortgage Back Securities Index and US Asset Backed Securities Index. The index holds investment grade bonds. The ratings are based on S&P, Moody and Fitch.

Bloomberg US Corporate High Yield Bond Index: The index is a USD denominated, fixed rate, taxable corporate bond benchmark that tracks the performance of the non investment grade (high yield or "junk") corporate bond market in the United States.

Bloomberg US 1-3 year Government Bond Index: The index measures the performance of US dollar-denominated, fixed-rate debt issued by the US Treasury with remaining maturities of at least 1 year but less than 3 years.

FTSE Non-US Dollar World Government Bond Index: The index is a sub index of the FTSE World Government Bond Index (WGBI).

JP Morgan EMBI Plus Index: The index is a benchmark that tracks the total return performance of external currency denominated sovereign and quasi sovereign debt instruments from emerging market countries—specifically, fixed income securities denominated in US dollars (Brady bonds, loans, and Eurobonds).

MSCI All Country World Index: A global equity benchmark. It's managed by MSCI Inc. (formerly Morgan Stanley Capital International), a leading provider of investment indexes.

MSCI EAFE Index: The MSCI EAFE Index (Europe, Australasia, Far East) is a free float-adjusted market capitalization index that is designed to measure the equity market performance of developed markets, excluding the US & Canada. The MSCI EAFE Index consists of the following 22 developed market country indices: Australia, Austria, Belgium, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Israel, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland and the United Kingdom.

MSCI Emerging Markets (EM) Index: The index is a free float-adjusted market capitalization index that is designed to measure equity market performance in the global emerging markets. The MSCI Emerging Markets Index consists of the following 21 emerging market country indices: Brazil, Chile, China, Colombia, Czech Republic, Egypt, Hungary, India, Indonesia, Korea, Malaysia, Mexico, Morocco, Peru, Philippines, Poland, Russia, South Africa, Taiwan, Thailand, and Turkey.

Russell 1000® Index: Measures the performance of the large-cap segment of the US equity universe. It is a subset of the Russell 3000 Index and includes approximately 1,000 of the largest securities based on a combination of their market cap and current index membership. The Russell 1000 represents approximately 92% of the Russell 3000 Index.

Russell 1000® Growth Index: Measures the performance of the large-cap growth segment of the US equity universe. It includes those Russell 1000 companies with higher price-to book ratios and higher forecasted growth values.

Russell 2000® Value Index: Measures the performance of the small-cap value segment of the US equity universe. It includes those Russell 2000 companies with lower price-to-book ratios and lower forecasted growth values.

Russell 1000® Index: Measures the performance of the large-cap segment of the US equity universe. It is a subset of the Russell 3000 Index and includes approximately 1,000 of the largest securities based on a combination of their market cap and current index membership. The Russell 1000 represents approximately 92% of the Russell 3000 Index.

Russell 2000® Growth Index: Measures the performance of the small-cap growth segment of the US equity universe. It includes those Russell 2000 companies with higher price-to-book ratios and higher forecasted growth values.

Russell 3000® Index: The index is a capitalization weighted stock market index that tracks the 3,000 largest publicly traded companies in the United States, representing approximately 98% of the investable US equity market.

1. Markowitz, Harry. 1959. Portfolio Selection: Efficient Diversification of Investments. Cowles Foundation Monograph No. 16. New York: John Wiley & Sons, Inc.

2. Sharpe, William F. 1964. "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." Journal of Finance. 19:3, pp. 425– 42.

3. Treynor, J. L. (1962). "Toward a Theory of Market Value of Risky Assets." Unpublished manuscript. "Rough Draft" dated by Mr. Treynor to the fall of 1962. A final version was published in 1999, in Asset Pricing and Portfolio Performance. Robert A. Korajczyk (editor) London: Risk Books, pp. 15–22.

4. "Fifty years of broad-based international data: What have we learned for asset allocation?", December 2021, Keith Gustafson and Christopher O'Neill, Mesirow Fiduciary Solutions white paper available on website.

5. "The Most Effective Portfolio Inflation Hedges", April 2022, Keith Gustafson and Christopher O'Neill, Mesirow Fiduciary Solutions white paper, available on website.

6. Multicollinearity produces a situation where there is more than one efficient portfolio for a given risk and return. For instance, the Russell 1000 consists of 50% Russell 1000 Growth and 50% Russell 1000 Value, so including all three results in an indeterminate solution, where in technical terms the covariance matrix is not positive semi-definite.